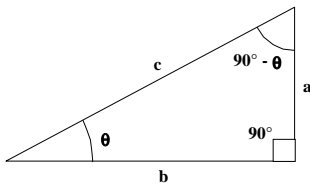


QUADRANTS & SIGNS OF FUNCTIONS

sin & csc pos. others neg.	All positive
II	I
III	IV
tan & cot pos. others neg.	cos & sec pos. others neg.

RIGHT-ANGLE TRIANGLE RELATIONSHIPS



a = opposite side
b = adjacent side
c = hypotenuse

$$\sin \theta = \frac{\text{side opposite } \theta}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos \theta = \frac{\text{side adjacent to } \theta}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan \theta = \frac{\text{side opposite } \theta}{\text{side adjacent to } \theta} = \frac{a}{b}$$

Pythagorean theorem: $c^2 = a^2 + b^2$

FUNDAMENTAL IDENTITIES

- $\tan x = \frac{\sin x}{\cos x}$
- $\sec x = \frac{1}{\cos x}$
- $\csc x = \frac{1}{\sin x}$
- $\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$
- $\cos^2 x + \sin^2 x = 1$
- $1 + \tan^2 x = \sec^2 x$
- $\cot^2 x + 1 = \csc^2 x$
- $\cos(\pi/2 - x) = \sin x$
- $\cos(\pi/2 + x) = -\sin x$
- $\sin(\pi/2 - x) = \cos x$
- $\sin(\pi/2 + x) = \cos x$
- $\tan(\pi/2 - x) = \cot x$

OPPOSITE-ANGLE IDENTITIES

- $\cos(-x) = \cos x$
- $\sin(-x) = -\sin x$
- $\tan(-x) = -\tan x$
- $\sec(-x) = \sec x$
- $\csc(-x) = -\csc x$
- $\cot(-x) = -\cot x$

ADDITION LAWS

$$1. \cos(x \pm y) = \cos x \cos y \mp \sin x \sin y \quad 2. \sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$3. \tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

DOUBLE-ANGLE IDENTITIES

- $\cos 2x = \cos^2 x - \sin^2 x$
- $\cos 2x = 2 \cos^2 x - 1$
- $\cos 2x = 1 - 2 \sin^2 x$
- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- $\sin 2x = 2 \sin x \cos x$
- $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

HALF-ANGLE IDENTITIES

- $\cos \frac{x}{2} = \pm \sqrt{\frac{1 + \cos x}{2}}$
 { + if x/2 is in quadrant I or IV
 - if x/2 is in quadrant II or III
- $\sin \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{2}}$
 { + if x/2 is in quadrant I or II
 - if x/2 is in quadrant III or IV
- $\tan \frac{x}{2} = \pm \sqrt{\frac{1 - \cos x}{1 + \cos x}}$
 { + if x/2 is in quadrant I or III
 - if x/2 is in quadrant II or IV
- $\tan \frac{x}{2} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x} = \csc x - \cot x$

PRODUCT IDENTITIES

- $2 \cos x \cos y = \cos(x - y) + \cos(x + y)$
- $2 \sin x \sin y = \cos(x - y) - \cos(x + y)$
- $2 \sin x \cos y = \sin(x + y) + \sin(x - y)$
- $2 \cos x \sin y = \sin(x + y) - \sin(x - y)$
- $\cos m x \cos n x = \cos(m + n)x + \cos(m - n)x$

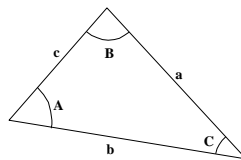
SUM IDENTITIES

- $\cos x + \cos y = 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$
- $\cos x - \cos y = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$
- $\sin x + \sin y = 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$
- $\sin x - \sin y = 2 \sin\left(\frac{x-y}{2}\right) \cos\left(\frac{x+y}{2}\right)$

REDUCTION IDENTITY

1. $a \sin x + b \cos x = \sqrt{a^2 + b^2} \sin(x + y)$, where y is chosen so that $\cos y = \frac{a}{\sqrt{a^2 + b^2}}$ and $\sin y = \frac{b}{\sqrt{a^2 + b^2}}$

PLANE TRIANGLE RELATIONSHIPS



Law of sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Law of cosines: $c^2 = a^2 + b^2 - 2ab \cos C$

$C = \arccos\left(\frac{a^2 + b^2 - c^2}{2ab}\right)$

Law of tangents: $\frac{a+b}{a-b} = \frac{\tan \frac{1}{2}(A+B)}{\tan \frac{1}{2}(A-B)}$

INVERSE TRIGONOMETRIC FUNCTIONS

Function	Domain	Range	Quadrants
$y = \cos^{-1} x$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$	I and II
$y = \sin^{-1} x$	$-1 \leq x \leq 1$	$-\pi/2 \leq y \leq \pi/2$	I and IV
$y = \tan^{-1} x$	all reals	$-\pi/2 < y < \pi/2$	I and IV
$y = \sec^{-1} x$	$x \geq 1$ or $x \leq -1$	$0 \leq y \leq \pi, y \neq \pi/2$	I and II
$y = \csc^{-1} x$	$x \geq 1$ or $x \leq -1$	$-\pi/2 \leq y \leq \pi/2, y \neq 0$	I and IV
$y = \cot^{-1} x$	all reals	$0 < y < \pi$	I and II

PRINCIPAL VALUES FOR INVERSE TRIGONOMETRIC FUNCTIONS

Principal values for $x \geq 0$	Principal values for $x < 0$
$0 \leq \sin^{-1} x \leq \pi/2$	$-\pi/2 \leq \sin^{-1} x < 0$
$0 \leq \cos^{-1} x \leq \pi/2$	$\pi/2 < \cos^{-1} x \leq \pi$
$0 \leq \tan^{-1} x < \pi/2$	$-\pi/2 < \tan^{-1} x < 0$
$0 < \cot^{-1} x \leq \pi/2$	$\pi/2 < \cot^{-1} x < \pi$
$0 \leq \sec^{-1} x < \pi/2$	$\pi/2 < \sec^{-1} x \leq \pi$
$0 < \csc^{-1} x \leq \pi/2$	$-\pi/2 \leq \csc^{-1} x < 0$

INVERSE IDENTITIES (ASSUMING PRINCIPAL VALUES ARE USED)

- $\sin^{-1} x + \cos^{-1} x = \pi/2$
- $\tan^{-1} x + \cot^{-1} x = \pi/2$
- $\sec^{-1} x + \csc^{-1} x = \pi/2$
- $\csc^{-1} x = \sin^{-1}(1/x)$
- $\sec^{-1} x = \cos^{-1}(1/x)$
- $\cot^{-1} x = \tan^{-1}(1/x)$
- $\sin^{-1}(-x) = -\sin^{-1} x$
- $\cos^{-1}(-x) = \pi - \cos^{-1} x$
- $\tan^{-1}(-x) = -\tan^{-1} x$
- $\cot^{-1}(-x) = \pi - \cot^{-1} x$
- $\sec^{-1}(-x) = \pi - \sec^{-1} x$
- $\csc^{-1}(-x) = -\csc^{-1} x$

COMPLEX IDENTITIES

$\cos w = \frac{e^{iw} + e^{-iw}}{2}$ $i \sin w = \frac{e^{iw} - e^{-iw}}{2}$ Euler Identity: $e^{iw} = \cos w + i \sin w$

QUADRATIC FORMULA

Solution to $ax^2 + bx + c = 0$: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

EXACT VALUES

Angle (deg)	Angle (rad)	cos(x)	sin(x)	tan(x)
0°	0	1	0	0
30°	$\pi/6$	$\sqrt{3}/2$	1/2	$\sqrt{3}/3$
45°	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60°	$\pi/3$	1/2	$\sqrt{3}/2$	$\sqrt{3}$
90°	$\pi/2$	0	1	$\pm\infty$
180°	π	-1	0	0
270°	$3\pi/2$	0	-1	$\pm\infty$